1. The demand function for a firm's product is given by $q = \dfrac{40\sqrt{5y+6p_s}}{2p+5}$, where

- q is the monthly demand for the firm's product, measured in 1000's of units;
- y is the average monthly disposable income in the market for the firm's product measured in 1000s of dollars;
- p_s is the average price of substitutes for the firm's product, measured in dollars;
- p is the price of the firm's product, also measured in dollars.

(a) (6 pts) Find $\partial q/\partial y$, $\partial q/\partial p_s$ and $\partial q/\partial p$ when the monthly income is \$3000 and the prices are $p_s = 11$ and $p = 10$. Round your (final) answers to two decimal places.

Remember: if monthly income is \$3000, then $y = 3$.

$$\frac{\partial q}{\partial y} = \frac{\partial}{\partial y}\left(\frac{40}{2p+5}(5y+6p_s)^{1/2}\right) = \frac{40}{2p+5}\cdot\frac{1}{2}(5y+6p_s)^{-1/2}\cdot 5 = \frac{100(5y+6p_s)^{-1/2}}{2p+5},$$

so $$\left.\frac{\partial q}{\partial y}\right|_{\substack{y=3\\p_s=11\\p=10}} = \frac{100(81)^{-1/2}}{25} = \frac{4}{9} \qquad (\approx 0.444).$$

$$\frac{\partial q}{\partial p_s} = \frac{\partial}{\partial p_s}\left(\frac{40}{2p+5}(5y+6p_s)^{1/2}\right) = \frac{40}{2p+5}\cdot\frac{1}{2}(5y+6p_s)^{-1/2}\cdot 6 = \frac{120(5y+6p_s)^{-1/2}}{2p+5},$$

so $$\left.\frac{\partial q}{\partial p_s}\right|_{\substack{y=3\\p_s=11\\p=10}} = \frac{120(81)^{-1/2}}{25} = \frac{8}{15} \qquad (\approx 0.533).$$

$$\frac{\partial q}{\partial p} = \frac{\partial}{\partial p}\left(40\sqrt{5y+6p_s}\right)(2p+5)^{-1} = \left(40\sqrt{5y+6p_s}\right)(-1)(2p+5)^{-2}\cdot 2 = -80\sqrt{5y+6p_s}(2p+5)^{-2},$$

so $$\left.\frac{\partial q}{\partial p}\right|_{\substack{y=3\\p_s=11\\p=10}} = \frac{-80\sqrt{81}}{25^2} = -\frac{144}{125} \qquad (= -1.152).$$

(b) (4 pts) Use *linear approximation* and your answer to (a) to ***estimate*** the change in monthly demand for the firm's product if the price of the firm's product decreases to \$9 and the price of substitutes decreases to \$10, but income remains fixed.

If income remains fixed, then $\Delta y = 0$, and the changes in the prices are $\Delta p_s = \Delta p = -1$, so

$$\Delta q \approx \left.\frac{\partial q}{\partial p_s}\right|_{\substack{y=3\\p_s=11\\p=10}}\cdot\Delta p_s + \left.\frac{\partial q}{\partial p}\right|_{\substack{y=3\\p_s=11\\p=10}}\cdot\Delta p = \frac{8}{15}\cdot(-1) + \left(-\frac{144}{125}\right)\cdot(-1) = \frac{144}{125} - \frac{8}{15} = \frac{232}{375} \qquad (\approx 0.619),$$

so demand will increase by about 619 units (remember that q is measured in 1000s of units).

3. (10 pts) Find the critical point(s) and critical value(s) of the function

$$f(x,y) = 3x^2 + y^3 - 6xy + 5,$$

and use the second derivative test to classify the critical value(s) as relative minimum value(s), relative maximum value(s) or neither.

Critical points:

$$f_x = 0 \implies 6x - 6y = 0$$
$$f_y = 0 \implies 3y^2 - 6x = 0$$

The equation $f_x = 0$ implies that $x = y$, and substituting this into the equation $f_y = 0$ gives

$$3y^2 - 6y = 0 \implies 3y(y-2) = 0,$$

so the critical y-values are $y_1 = 0$ and $y_2 = 2$ and the critical points are $(0,0)$ and $(2,2)$.

Second derivative test:

$$f_{xx} = 6, \; f_{yy} = 6y \;\text{ and }\; f_{xy} = -6 \quad \text{so} \quad D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = 36y - 36$$

At the critical point $(0,0)$, we have $D(0,0) = -36 < 0$, so $f(0,0) = 5$ is neither a minimum nor a maximum value.

At the critical point $(2,2)$, we have $D(0,0) = 36 > 0$ and $f_{xx}(2,2) = 6 > 0$, so $f(2,2) = 1$ is a relative minimum value.