1. (6 pts) $\displaystyle\int_0^3 \frac{2x}{\sqrt{7x+4}}\,dx = 2\left(\frac{2(7x-8)\sqrt{7x+4}}{147}\Bigg|_0^3\right) = 2\left(\frac{26\sqrt{25}}{147} - \frac{-16\sqrt{4}}{147}\right) = \frac{108}{49}$ (≈ 2.2041),

using the formula $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C,$ with $a=4$ and $b=7$.

2. (6 pts) Find the present value of a continuous income stream that pays at the annual rate $f(t) = 1000t$ for $T = 10$ years, assuming that the interest rate is $r = 3\%$.

$$PV = \int_0^T f(t)e^{-rt}\,dt = \int_0^{10} 1000te^{-0.03t}\,dt = 1000\left(\frac{e^{-0.03t}}{(-0.03)^2}(-0.03t-1)\Bigg|_0^{10}\right) \approx 41040.35,$$

using the formula $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C$, with $a = -0.03$.

3. (4 pts) Find the function $y = f(x)$ that satisfies (i) $\dfrac{dy}{dx} = xy^2$ and (ii) $f(0) = \dfrac{1}{2}$.

Separate: $\dfrac{dy}{dx} = xy^2 \implies \dfrac{dy}{y^2} = x\,dx.$

Integrate: $\displaystyle\int \frac{dy}{y^2} = \int x\,dx \implies -\frac{1}{y} = \frac{x^2}{2} + C.$

Solve for C: $f(0) = \dfrac{1}{2} \implies -\dfrac{1}{1/2} = \dfrac{0^2}{2} + C \implies C = -2.$

Solve for y: $-\dfrac{1}{y} = \dfrac{x^2}{2} - 2 = \dfrac{x^2-4}{2} \implies \dfrac{1}{y} = \dfrac{4-x^2}{2} \implies y = \dfrac{2}{4-x^2}.$

Comment: You can also solve for y first and then solve for C.

4. (4 pts) Find the indicated partial derivatives of the function $f(x,y) = 3x^2\ln(2x+5y)$. Clean up your answer.

$\dfrac{\partial f}{\partial x} = 6x\ln(2x+5y) + 3x^2\cdot\dfrac{2}{2x+5y} = 6x\ln(2x+5y) + \dfrac{6x^2}{2x+5y}$ (product rule and chain rule for $\ln(2x+5y)$).

$\dfrac{\partial f}{\partial y} = 3x^2\cdot\dfrac{5}{2x+5y} = \dfrac{15x^2}{2x+5y}$ (chain rule).