1. (6 pts) $\displaystyle\int_1^3 4x(x^2-1)^{2/3}\,dx = \int_0^8 2u^{2/3}\,du = \left.\frac{2u^{5/3}}{5/3}\right|_0^8 = \frac{6}{5}\left(8^{5/3}-0^{5/3}\right) = \frac{192}{5} \quad (=38.4)$

Make the substitution: $u = x^2-1$, so $du = 2x\,dx$ and $4x\,dx = 2du$. Also, the limits of integration change: $x=1 \implies u=0$ and $x=3 \implies u=8$.

2. (7 pts) A firm's marginal cost function is given by $dc/dq = 5+0.02q$, where q is the number of deluxe widgets the firm produces and cost is measured in \$1000s. Find the ***total change*** in the firm's cost if output increases from 10 widgets to 15 widgets.

According to the fundamental theorem of calculus:

$$c(15)-c(10) = \int_{10}^{15}\frac{dc}{dq}\,dq = \int_{10}^{15} 5+0.02q\,dq = 5q+\left.\frac{0.02q^2}{2}\right|_{10}^{15} = (75+2.25)-(50+1) = 26.25$$

(I.e., cost increases by \$26,250.00)

3. (7 pts) Find the Gini coefficient of inequality for the nation whose wealth distribution curve is

$$f(x) = 0.8x^4+0.2x^3.$$

(In this case, $f(x)\times 100\%$ is the percentage of the national wealth owned by the poorest $x\times 100\%$ of the population.)

The Gini coefficient (γ) is defined by the first expression on the left below, but can be computed using any of the other equivalent expressions:

$$\gamma = \frac{\displaystyle\int_0^1 x-f(x)\,dx}{\displaystyle\int_0^1 x\,dx} = \frac{\displaystyle\int_0^1 x\,dx - \int_0^1 f(x)\,dx}{\frac{1}{2}} = 2\left(\frac{1}{2}-\int_0^1 f(x)\,dx\right) = 1-2\int_0^1 f(x)\,dx.$$

For the given wealth distribution curve, we have

$$\gamma = 1-2\int_0^1 0.8x^4+0.2x^3\,dx = 1-2\left(\frac{0.8x^5}{5}+\left.\frac{0.2x^4}{4}\right|_0^1\right) = 1-2\big((0.16+0.05)-(0+0)\big) = 0.58.$$