1. (6 pts) $\displaystyle\int \frac{3x^2+8x+1}{2x}\,dx = \int \frac{3}{2}x + 4 + \frac{1}{2}\cdot\frac{1}{x}\,dx = \frac{3}{4}x^2 + 4x + \frac{1}{2}\ln|x| + C$

2. (7 pts) $\displaystyle\int \frac{5x}{\sqrt{x^2+1}}\,dx = \frac{5}{2}\int u^{-1/2}\,du = \frac{5}{2}\cdot\frac{u^{1/2}}{1/2} + C = 5u^{1/2} + C = 5(x^2+1)^{1/2} + C,$

making the substitution $u = x^2+1$, $du = 2x\,dx$ (so $5x\,dx = \frac{5}{2}\,du$).

3. (7 pts) The marginal propensity to consume for a nation is given by

$$\frac{dC}{dY} = \frac{8Y+15}{9Y+1},$$

where C is the nation's annual consumption and Y is annual income, both measured in trillions of dollars. Find the total change in the nation's consumption if national income increases from $Y = 5$ trillion dollars to $Y = 7$ trillion dollars.

(a) integrate dC/dY (using long division to simplify the integrand and the substitution $u = 9Y+1$, $du = 9dY$ for the second term):

$$\int \frac{8Y+15}{9Y+1}\,dY = \int \frac{8}{9} + \frac{127/9}{9Y+1}\,dY = \frac{8}{9}Y + \frac{127}{81}\ln|9Y+1| + K.$$

This means that the consumption function is $C = \frac{8}{9}Y + \frac{127}{81}\ln|9Y+1| + K$, for some unknown constant K.

(b) Calculate $C(7) - C(5)$ and note that the constant K cancels when we subtract:

$$C(7) - C(5) = \overbrace{\left(\frac{8}{9}\cdot 7 + \frac{127}{81}\ln|9\cdot 7+1| + \cancel{K}\right)}^{C(7)} - \overbrace{\left(\frac{8}{9}\cdot 5 + \frac{127}{81}\ln|9\cdot 5+1| + \cancel{K}\right)}^{C(5)} \approx 2.2956.$$

I.e., when income increases from \$5 trillion to \$7 trillion, consumption increases by about \$2.2956 trillion.

Note: The nation is spending more than it is taking in.